Exercise 9

Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1.

$$\sqrt[4]{1+2x} \approx 1 + \frac{1}{2}x$$

Solution

Plugging in $x = 0$ to the function yields $\sqrt[4]{1+2(0)} = 1$, so $(0, 1)$ is the point on the curve that the tangent line goes through. Taking the derivative of the function yields

$$\frac{d}{dx}\sqrt[4]{1+2x} = \frac{d}{dx}(1+2x)^{1/4} = \frac{1}{4}(1+2x)^{-3/4} \cdot \frac{d}{dx}(1+2x) = \frac{1}{4}(1+2x)^{-3/4} \cdot 2 = \frac{1}{2}(1+2x)^{-3/4}.$$

Set $x = 0$ to get the slope of the tangent line.

$$\frac{d}{dx}\sqrt[4]{1+2x}\bigg|_{x=0} = \frac{1}{2}[1+2(0)]^{-3/4} = \frac{1}{2}$$

Use the point-slope formula to get the equation of this line.

$$y - 1 = \frac{1}{2}(x - 0)$$

$$y - 1 = \frac{1}{2}x$$

$$y = \frac{1}{2}x + 1$$

As a result, the linearization to $\sqrt[4]{1+2x}$ at 0 is

$$L(x) = \frac{1}{2}x + 1.$$

Now find the values of x for which the linear approximation is accurate to within 0.1.

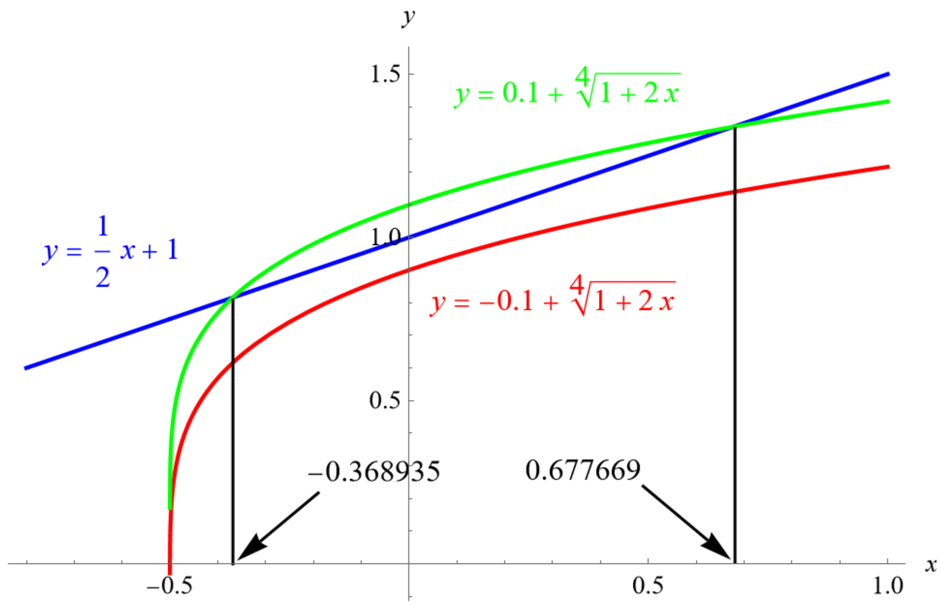
$$\left|\sqrt[4]{1+2x} - \left(\frac{1}{2}x + 1\right)\right| < 0.1$$

$$\left|\left(\frac{1}{2}x + 1\right) - \sqrt[4]{1+2x}\right| < 0.1$$

$$-0.1 < \left(\frac{1}{2}x + 1\right) - \sqrt[4]{1+2x} < 0.1$$

$$-0.1 + \sqrt[4]{1+2x} < \frac{1}{2}x + 1 < 0.1 + \sqrt[4]{1+2x}$$

Plot each of these functions versus x.



The linear approximation stays between the curves for

$$-0.368935 < x < 0.677669.$$

This is the interval that the linear approximation is accurate to within 0.1.